

L 10/12/2011

FE Sem-I (Rev)

Applied Maths I

PR-Oct. (1) 101

Con. 5840-11.

MP-3355

[Total Marks : 100

(3 Hours)

- N.B. :** (1) Question No. 1 is compulsory.  
 (2) Attempt any four questions from the remaining questions.  
 (3) Figures to the right indicate full marks.

1. (a) If  $\arg(z + 1) = \frac{\pi}{6}$  and  $\arg(z - 1) = \frac{2\pi}{3}$ , find  $z$ , a complex number. 5
- (b) Prove that  $\tanh^{-1}(\sin \theta) = \cosh^{-1}(\sec \theta)$ . 5
- (c) Prove that the real part of  $(1+i\sqrt{3})^{(1+i\sqrt{3})}$  is  $2e^{-\frac{1}{\sqrt{3}}} \cos\left(\frac{\pi}{3} + \sqrt{3} \cdot \log 2\right)$ . 5
- (d) Test the convergence of  $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} + \dots (x > 0, x \neq 1)$ . 5
2. (a) Prove that  $\frac{a+ib}{1+c} = \frac{1+iz}{1-iz}$ , if  $a^2 + b^2 + c^2 = 1$  and  $b + ic = (1 + a)z$ . 6
- (b) Find the roots  $\alpha, \alpha^2, \alpha^3, \alpha^4$  of the equation  $x^5 - 1 = 0$  and show that  $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$  6
- (c) Prove that  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ , if  $u = f(e^{y-z}, e^{z-x}, e^{x-y})$ . 8
- (a) Prove that  $\arg z_1 - \arg z_2 = \frac{\pi}{2}$ , if  $|z_1 + z_2| = |z_1 - z_2|$ ,  $z_1, z_2$  being complex numbers. 6
- (b) Prove that  $\alpha^n + \beta^n = 2 \cos n\theta \operatorname{cosec}^n \theta$ , if  $\alpha, \beta$  are the roots of the equation  $z^2 \sin^2 \theta - z \sin 2\theta + 1 = 0$  6
- (c) Show that  $\tan^{-1} \left| \frac{x-a}{x+a} \right| = \frac{i}{2} \log \frac{x}{a}$ . 8
4. (a) Prove that  $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$ , if  $\cos^{-1}(y/b) = \log\left(\frac{x}{n}\right)^n$ . 6
- (b) Show that  $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$ , if  $z = \tan(y + ax) + (y - ax)^{3/2}$ . 6
- (c) Prove that  $2x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 4x$ , if  $f(xy^2, z - 2x) = 0$ . 8
5. (a) Separate into real and imaginary parts  $\operatorname{cosec}^{-1}\left(\frac{3i}{4}\right)$ . 6

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(b) Prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ , if  $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ . 6

(c) Examine the function  $f(x, y) = y^2 + 4xy + 3x^2 + x^3$  for extreme values. 8

6. (a) Find  $x$ , if  $\bar{a} = x\mathbf{i} + 12\mathbf{j} - \mathbf{k}$ ,  $\bar{b} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ ,  $\bar{c} = \mathbf{i} + \mathbf{k}$  are coplanar. Also find unit vector in the direction of vector  $\bar{a}$ . 6

(b) Prove that  $\log \sec x = \frac{1}{2} x^2 + \frac{1}{12} x^4 + \frac{1}{45} x^6 + \dots$  6

(c) Evaluate  $\lim_{x \rightarrow 0} \frac{e^x \sin x - x - x^2}{x^2 + x \log(1-x)}$ . 8

7. (a) Prove that,  $\frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} \cdot \frac{dz}{dx} = \frac{\partial f}{\partial x} \cdot \frac{\partial \phi}{\partial y}$ , if  $f(x, y) = 0$  and  $\phi(y, z) = 0$ . 6

(b) Find  $(1.04)^{3.01}$ , by using the theory of approximation. 6

(c) Prove that  $[\bar{b} \times \bar{c} \quad \bar{a} \times \bar{c} \quad \bar{a} \times \bar{b}] = [\bar{a} \quad \bar{b} \quad \bar{c}]^2$  8