F. E. (Sem I) All Branches
Applied Mathematics - I

Con. 3383-11. RK-5046

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions from the remaining **six** questions.

Q.1 a) If z_1 and z_2 are two complex numbers such that $|z_1+z_2|=|z_1-z_2|$; prove that the difference of their amplitude is $\frac{\pi}{2}$. 05

b) If $z=\sin^{-1}(x-y)$, $x=3t$, $y=4t^3$ then prove that $\frac{dz}{dt}=\frac{3}{\sqrt{1-t^2}}$ 05

c) Find the value of n for which the vector $r^n\bar{r}$ is solenoidal, where $\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$ 05

d) Find n^{th} order derivative of $y=\sin 2x \sin 3x \cos 4x$ 05

Q.2 a) If α and β are the roots of the equation $x^2-2x+4=0$ then show that $\alpha^n+\beta^n=2^{n+1}\cos\left(\frac{n\pi}{3}\right)$ and hence find the value of $\alpha^{15}+\beta^{15}$ 06

b) If $x=\cosh\left(\frac{1}{m}\log y\right)$, prove that $(x^2-1)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$ 06

c) If $v=x\log(x+r)-r$ where $r^2=x^2+y^2$, prove that $\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=\frac{1}{x+r}$ 08

Q.3 a) If $z=x^2\tan^{-1}\left(\frac{y}{x}\right)-y^2\tan^{-1}\left(\frac{x}{y}\right)$, prove that $\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}=\frac{x^2-y^2}{x^2+y^2}$ 06

b) If ω is a complex cube root of unity, prove that $(1-\omega)^6=-27$ 06

c) Find the constants a, b such that the surfaces $5x^2-2yz-9x=0$ and $ax^2y+bz^3=4$ cut orthogonally at $(1,-1,2)$ 08

[TURN OVER

Q.4 a) Prove that $\cos^{-1}[\tanh(\log x)] = \pi - 2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots\dots\dots\right)$ 06

b) If $x^2 = au + bv, y^2 = au - bv$, prove that $\left(\frac{\partial u}{\partial x}\right)_y \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2} = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u$, where a, b are constants. 06

c) Prove that $\cos^6\theta + \sin^6\theta = \frac{1}{8}(3\cos 4\theta + 5)$ 08

Q.5 a) If $0 < a < b$, prove that $\left(1 - \frac{a}{b}\right) < \log\frac{b}{a} < \left(\frac{b}{a} - 1\right)$. Hence prove that 06

$\frac{1}{6} < \log(1.2) < \frac{1}{5}$ and $\frac{1}{2} < \log 2 < 1$

b) Prove that $\tanh^{-1} x = \sinh^{-1}\frac{x}{\sqrt{1-x^2}}$ 06

c) At a distance 120 feet from the foot of a tower, the elevation of its top is 60°. If the possible error in measuring the distance and elevation are 1 inch and 1 minute respectively, find the approximate error in the calculated height of the tower. 08

Q.6 a) If $\tan(x + iy) = \alpha + i\beta$, show that $\frac{1 - \alpha^2 - \beta^2}{1 + \alpha^2 + \beta^2} = \frac{\cos 2x}{\cosh 2y}$ 06

b) If $z = \log(x^2 + y^2) + \frac{x^2 + y^2}{x + y} - 2\log(x + y)$, find the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ 06

c) If $\overline{A} = (\sin t)\hat{i} + (\cos t)\hat{j} + t\hat{k}$, $\overline{B} = (\cos t)\hat{i} - (\sin t)\hat{j} - 3\hat{k}$, $\overline{C} = 2\hat{i} + 3\hat{j} - \hat{k}$, 08

find $\frac{d}{dt}\left[\overline{A} \times (\overline{B} \times \overline{C})\right]$ at $t = 0$.

Q.7 a) If $(a + ib)^p = m^{x+iy}$, prove that $\frac{y}{x} = \frac{2\tan^{-1}\frac{b}{a}}{\log(a^2 + b^2)}$ 06

b) Prove that $\lim_{x \to 0}\left(\frac{a}{x} - \cot\frac{x}{a}\right) = 0$ 06

c) Find the extreme values of $u = x^3 + 3xy^2 - 3x^2 - 3y^2 + 7$, if any. 08
