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FE Semr II (Rev)
Applied maths II

AGJ 2nd half (q) 30

Con. 5822-11.

MP-2522

(3 Hours)

[Total Marks : 100

- N.B. :** (1) Question No. 1 is compulsory.
 (2) Out of remaining questions, attempt any four questions.
 (3) In all five questions to be attempted.
 (4) Answer to each new question to be started on a fresh page.
 (5) Figures in brackets indicate full marks.

Q.1 a) Prove that $\sqrt{\frac{3}{2}-x} \sqrt{\frac{3}{2}+x} = \left(\frac{1}{4}-x^2\right) \pi \sec \pi x$ provided $-1 < 2x < 1$ (5)

b) Solve $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = e^x + 1$ (5)

c) Find the length of the curve $y = \log(e^x + 1) - \log(e^x - 1)$ from $x = 1$ to $x = 2$ (5)

d) Find the area bounded by the curves $xy = 2$, $4y = x^2$ and $y = 4$ (5)

Q.2 a) Change the order of integration in $\int_0^a \int_0^{b/(b+x)} f(x, y) dx dy$ (6)

b) Solve by the method of variation of parameters $(D^2 + 3D + 2)y = \frac{1}{1+e^x}$ (6)

c) Solve $\frac{dy}{dx} = 2 + \sqrt{xy}$ with $x_0 = 1.2$ $y_0 = 1.6403$ by Euler's modified formula for $x = 1.6$ correct the four places of decimals by taking $h = 0.2$. (8)

Q.3 a) Evaluate $\int_0^e \int_0^{\log y} \int_1^{e^x} \log z dx dy dz$ (6)

b) Change to polar coordinates and evaluate $\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \log(x^2 + y^2) dx dy$ (6)

c) Solve the differential equation $\frac{dy}{dx} = x + y^2$, $y(0) = 1$ for the interval $(0, 0.2)$ in steps of $h = 0.1$ by using Raunge-Kutta method of fourth order (8)

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Q.4 a) Solve $(D^2 - 2D + 1)y = xe^x \sin x$ (6)

b) Evaluate $\int_0^{\infty} \frac{e^{-x}}{x} \left(a - \frac{1}{x} + \frac{1}{x} e^{-ax} \right) dx$ (6)

c) Solve $(1+2x)^2 \frac{d^2y}{dx^2} - 6(1+2x) \frac{dy}{dx} + 16y = 8(1+2x)^2$ (8)

Q.5 a) Solve $\frac{dy}{dx} = e^{x-y} (e^x - e^y)$ (6)

b) Solve $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = \frac{x^3}{x^2 + 1}$ (6)

c) Find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$ (8)

Q.6 a) Find the mass of the lamina bounded by the curves $ay^2 = x^3$ and the line $y = x$, if the density at a point varies as the distance of the point from the x axis. (6)

b) Using duplication formula prove that $B(n, n) \cdot B\left(n + \frac{1}{2}, n + \frac{1}{2}\right) = \frac{\pi}{n} 2^{1-4n}$ (6)

c) Solve $(D^2 + 1)y = x^2 \sin 3x$ (8)

Q.7 a) Evaluate $\iint (x^2 + y^2) dx dy$ over the area of the triangle whose vertices are $(0, 1)$, $(1, 1)$ and $(1, 2)$. (6)

b) Solve $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$ (6)

c) Evaluate $\int_0^{\infty} \frac{e^{-\beta x} \sin 2x}{x} dx$ and hence deduce that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$ (8)

* Good Luck *