FE II (All Branches)(Rev)
Applied Mathematics II

Con. 3096–11. **RK–1122**

(3 Hours) **[Total Marks : 100**

Note: 1. Question number one is compulsory.

2. Solve any four out of the remaining six questions from Q2 to Q7.

3. Draw neat sketches wherever necessary.

Q.1. [20]

a) Solve $\frac{dy}{dx} + \frac{y\log y}{x-\log y} = 0$

b) Prove that $\int_0^1 (x\log x)^4\,dx = \frac{4!}{5^5}$.

c) Use DUIS to prove that, $\int_0^\infty \frac{\log(1+ax^2)}{x^2}\,dx = \pi\sqrt{a}$,

d) Find the length of arc of $r = a(1-\cos\theta)$ lying outside the circle $r = a\cos\theta$

Q.2.

a) Use method of variation of parameters to solve the equation , $(D^2 - 2D + 2)y = e^x \tan x$ [6]

b) Use Euler's modified method to find the value of y satisfying the equation $\frac{dy}{dx} = \log(x+y)$, [6] *for* $x = 1.2$ and $x = 1.4$ *correct to three decimals by taking* $h = 0.2$ *and* $y(1) = 2$.

(c) Change the order of integration and evaluate $\int_0^\infty \int_0^x x e^{\frac{-x^2}{y}}\,dy\,dx$. [8]

Q.3 a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$ [6]

b) Change to polar co-ordinates and evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x\,dx\,dy}{\sqrt{x^2+y^2}}$. [6]

c) Solve the differential equation $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$ for the interval (0, 1) choosing h= 0.5 by [8] using Runge- kutta Method of fourth order.

[TURN OVER

Q.4 a) Solve $(D^2 - 4D + 4)\, y = x^2 + e^x + \cos 2x$ [6]

b) Evaluate $\int_0^{\pi} \frac{dx}{a+b\cos x} : a > 0, b > 0$ (i) $\int_0^{\pi} \frac{dx}{(a+b\cos x)^2} = \frac{\pi a}{(a^2-b^2)^{3/2}}$

(ii) $\int_0^{\pi} \frac{\cos x\, dx}{(a+b\cos x)^2} = \frac{-\pi b}{(a^2-b^2)^{3/2}}$. [6]

c) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$ [8]

Q.5 a) Find the length of the loop of the curve $3ay^2 = x(x-a)^2$ [6]

b) Solve $\frac{dy}{dx} = x^3y^3 - xy.$ [6]

c) Evaluate $\int\int\int \frac{dxdydz}{(1+x+y+z)^3}$ *over the volume of tetrahedron bounded by planes* [8]

$x = 0, y = 0, z = 0, \text{and } x + y + z = 1.$

Q.6. a) Prove that $\int_0^{\infty} \frac{dx}{(e^x+e^{-x})^n} = \frac{1}{4}\beta\left(\frac{n}{2},\frac{n}{2}\right)$ & *hence evaluate* $\int_0^{\infty} sech^8 x dx.$ [6]

b) Solve $(D^2 - D - 2)\, y = 2\log x + 1/x + 1/x^2$ [6]

C. An electric circuit consists of an inductance L, a capacitance of capacity C and an e.m.f $E = E_0 Cos\omega t$,

so that the charge Q satisfies the differential equation $\frac{d^2Q}{dt^2} + \frac{Q}{CL} = \frac{E_0}{L} Cos\omega t$, If $\omega = \frac{1}{\sqrt{CL}}$ and

initially $Q = Q_0, at, t = 0, and current, i = i_0, at, t = 0$, find the charge Q at time t.

Q.7 a) Find the mass of the lamina bounded by the curve, $y^2 = ax$, $x^2 = ay$, Where density of the lamina at any point varies as the square of its distance from the origin. [6]

b) Change the order of integration $\int_0^a \int_{\sqrt{a^2-y^2}}^{y+a} f(x,y)\,dxdy$ [6]

c) I. *Evaluate* $\int_0^{\frac{\pi}{6}} \cos^3(3\theta)\sin^2(6\theta)\,d\theta$ [3]

II State and prove Duplication formula. [5]

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