

- N.B. : (1) Question No. 1 is compulsory.
 (2) Attempt any three questions from the remaining five.
 (3) Figures to the right indicate full marks.

1. (a) Prove that $\frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \cosh^2 x}}} = \cosh^2 x$ 3

(b) If $u = \log [\tan x + \tan y]$, prove that, 3
 $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} = 2.$

(c) If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1} x + \tan^{-1} y$, find $\frac{\partial(u,v)}{\partial(x,y)}$ 3

(d) Show that $\log[1 + \sin x] = x - \frac{x^2}{2} + \frac{x^3}{6} + \dots$ 3

(e) Show that every square matrix can be uniquely expressed as $P + iQ$, where P and Q are Hermitian matrices. 4

(f) Find n^{th} order derivative of $\frac{x^2+4}{(x-1)^2(2x+3)}$ 4

2. (a) Show that the roots of the equation $(x + 1)^6 + (x - 1)^6 = 0$ are given by 6

$$-i \cot \left[\frac{(2k+1)\pi}{12} \right], k = 0, 1, 2, 3, 4, 5.$$

(b) Reduce the following matrix into normal form and find its rank 6

$$\begin{bmatrix} 2 & -1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 3 & 3 & 3 & 1 \\ 1 & 4 & 2 & 0 \\ 0 & -4 & -1 & 2 \end{bmatrix}$$

(c) State and prove the Euler's theorem for a homogeneous function in two variables. 8

Hence find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ if $u = \frac{\sqrt{xy}}{\sqrt{x} + \sqrt{y}}$.

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3. (a) Test for consistency and solve if consistent -

$$\begin{aligned}x_1 - 2x_2 + x_3 - x_4 &= 2; & x_1 + 2x_2 + 2x_4 &= 1; \\4x_2 - x_3 + 3x_4 &= -1\end{aligned}$$

- (b) Find all the stationary values of $x^3 + 3xy - 15x^2 - 15y^2 + 72x$.

- (c) If $\tan\left(\frac{\pi}{4} + iv\right) = re^{i\theta}$, show that

$$(i) r = 1 \quad (ii) \tan \theta = \sinh 2v \quad (iii) \tan hv = \tan \frac{\theta}{2}.$$

4. (a) If $x = u + e^{-v} \sin u$, $y = v + e^{-u} \cos u$ find $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$ by using Jacobian.

- (b) Considering only the principal value, if $(1 + i \tan \alpha)^{1 + i \tan \beta}$ is real, prove that its value is $(\sec \alpha)^{\sec^2 \beta}$.

- (c) Solve the system of linear equation by Crout's method

$$x - y + 2z = 2; \quad 3x + 2y - 3z = 2; \quad 4x - 4y + 2z = 2.$$

5. (a) Expand $\cos^7 \theta$ in a series of cosines of multiples of θ .

- (b) Evaluate $\lim_{x \rightarrow 0} \left[\frac{1}{x^2} - \cot^2 x \right]$

- (c) If $y = (\sin^{-1} x)^2$, obtain $y_n(0)$.

6. (a) Show that the vectors are linearly dependent and find the relation between them :-

$$X_1 = [1, 2, -1, 0], \quad X_2 = [1, 3, 1, 2], \quad X_3 = [4, 2, 1, 0], \quad X_4 = [6, 1, 0, 1]$$

- (b) If $\frac{x^2}{1+u} + \frac{y^2}{2+u} + \frac{z^2}{3+u} = 1$,

Prove that,

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2 \left[x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} \right]$$

- (c) Fit a second degree parabolic curve to the following data :-

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9