16 17/5/12 FE Sem-I (Rev) A-Maths-I

Con. 3434-12. **GN-4925**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question **No. 1** is **compulsory.**
(2) Attempt any **four (04)** questions out of remaining **six (06)** questions.
(3) **Figures** to the **right** indicate **full** marks.

Q: 1. A) If $\text{Arg}(z+1) = \frac{\pi}{6}$ & $Arg(z-1) = \frac{2\pi}{3}$ find z. 5

B) Find n^{th} derivatie of $2^x \cdot \sin^2 x \cdot \cos^3 x$. 5

C) Expand $f(x) = x^4 - 3x^3 + 2x^2 - x + 1$ in powers of $(x-3)$. 5

D) Show that $\lim_{x \to \infty} \left(\frac{1^{1/x} + 2^{1/x} + 3^{1/x} + 4^{1/x}}{4} \right)^{4x} = 24$. 5

Q: 2. A) If $z = \log(x^2+y^2) + \frac{x^2+y^2}{x+y} - 2\log(x+y)$, 7

prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{x^2+y^2}{x+y}$

B) If $f(x), \emptyset(x), \varphi(x)$ are differentiable in $[a,b]$, show that 6
there exist a value c in (a,b) such that

$$\begin{vmatrix} f(a) & \emptyset(a) & \varphi(a) \\ f(b) & \emptyset(b) & \varphi(b) \\ f'(c) & \emptyset'(c) & \varphi'(c) \end{vmatrix} = 0$$

C) *If* $\tan(\alpha + i\beta) = e^{i\theta}$, *prove that*

1] $\alpha = \frac{n\pi}{2} + \frac{\pi}{4}$, *and* 3

2] $\beta = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$. 4

Q: 3. A) Show that $(1 - e^{i\theta})^{-1/2} + (1 - e^{-i\theta})^{-1/2} = \left(1 + \text{cosec}\left(\frac{\theta}{2}\right)\right)^{1/2}$. 7

B) If $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x}$ then show that c of C. M. V. T. 6
is H. M. of a & b where $a > 0, b > 0$.

C) *If* $y = \sin[\log(x^2 + 2x + 1)]$ *then prove that* 7
$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$.

Q: 4. A) Solve $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 = 0$. 7

B) If $u = lx + my$, $v = mx - ly$, prove that

1] $\left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v = \frac{l^2}{l^2+m^2}$ 3

2] $\left(\frac{\partial y}{\partial v}\right)_x \cdot \left(\frac{\partial v}{\partial y}\right)_u = \frac{l^2+m^2}{l^2}$. [illegible]

C) Show that $\bar{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ 7

is both solenoidal and irrotational.

Q: 5. A) Prove that $\tan 7\theta = \frac{7\tan\theta - 35\tan^3\theta + 21\tan^5\theta - \tan^7\theta}{1 - 21\tan^2\theta + 35\tan^4\theta - 7\tan^6\theta}$ 7

B) Test the convergence of $\sum \frac{2^n+1}{3^n+n}$ 6

C) If $x = \sqrt{vw},\ y = \sqrt{wu},\ z = \sqrt{uv}$, and φ is a function of x, y, z 7

prove that $u\frac{\partial\varphi}{\partial u} + v\frac{\partial\varphi}{\partial v} + w\frac{\partial\varphi}{\partial w} = x\frac{\partial\varphi}{\partial x} + y\frac{\partial\varphi}{\partial y} + z\frac{\partial\varphi}{\partial z}$

Q: 6. A) Prove that $\nabla \cdot \left\{\frac{f(r)}{r}\bar{r}\right\} = \frac{1}{r^2}\frac{d}{dr}[r^2 f(r)]$. 7

hence or otherwise prove that $\text{div}\,(r^n\bar{r}) = (n+3)r^n$.

B) Show that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1}b - \sin^{-1}a < \frac{b-a}{\sqrt{1-b^2}}$ hence show that

1] $\frac{\pi}{6} + \frac{\sqrt{3}}{15} < \sin^{-1}\left(\frac{3}{5}\right) < \frac{\pi}{6} + \frac{1}{8}$ 3

2] $\frac{\pi}{6} + \frac{1}{2\sqrt{3}} < \sin^{-1}\left(\frac{1}{4}\right) < \frac{\pi}{6} - \frac{1}{\sqrt{15}}$ 3

C) If $u = \frac{x^3y^3z^3}{x^3+y^3+z^3} + \log\left(\frac{xy+yz+zx}{x^2+y^2+z^2}\right)$ then prove that 7

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 6\frac{x^3y^3z^3}{x^3+y^3+z^3}$$

Q: 7. A) Find the extreme values of the function 7

$x^3 + 3xy^2 + 72x - 15x^2 - 15y^2$.

B) Examine the convergence of $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \cdots\cdots$ 6

C) If $\frac{(a+ib)^{x+iy}}{(a-ib)^{x-iy}} = \alpha + i\beta$ then find α & β. 7