FE | Sem-II (Rev) 14|6|2012
Applied mathematics-II

(Two papers due to re-exam) 14/06/2012 original paper

(3 Hours) GN-1018

[Total Marks 100

N.B.: 1. Question **No. 1** is compulsory.

2. Attempt **any four** questions from remaining six questions.

3. Draw sketches wherever necessary.

Q.1.a . Evaluate : $\int_0^1 (x\log x)^4\, dx$ (5)

b. Solve : $\frac{dr}{d\theta} = r\tan \; -- \; \frac{r^2}{\cos\theta}$ (5)

c. Evaluate: $\int_0^{a\sqrt{3}} \int_0^{\sqrt{x^2+a^2}} \frac{x\, dy\, dx}{y^2+x^2+a^2}$ (5)

d. Find by double integration the area enclosed by $y^2 = x^3$ and y = x (5)

Q.2.a. Solve $(4xy + 3y^2 - x)\, dx + x\,(x + 2y)\, dy = 0$ (6)

b. Change the order of integration $\int_0^a \int_{\sqrt{a^2-y^2}}^{y+a} f(x,y)\, dx\, dy$ (6)

c. Prove that $\int_0^\infty \frac{dx}{(e^x+e^{-x})^n} = \frac{1}{4}\, \beta\left(\frac{n}{2}, \frac{n}{2}\right)$ and hence evaluate $\int_0^\infty sech^6 x\, dx$. (8)

Q.3.a. Using Euler's method find approximate value of y at x=1 in five steps taking h=0.2 given $\frac{dy}{dx} = x + y$ & $y(0) = 1$. (6)

b. Evaluate $\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z}\, dz\, dy\, dx$ (6)

c. Evaluate by changing to polar coordinates $\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2 + y^2)\, dy dx$ (8)

Q.4.a. Show that $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)}\, dx = \frac{\pi}{2} \log(1+a)$ (6)

b. Evaluate $\int_R \int \frac{y\, dx\, dy}{(a-x)\sqrt{ax-y^2}}$ where R is the region bounded by $y^2 = ax$ & $y = x$. (6)

c. Solve by the method of variation of parameters $(D^2 - 2D + 2)y = e^x \tan x$ (8)

Q.5.a. Solve $(D^2 + 2)y = e^x \cos x + x^2 e^{3x}$ (6)

b. Using Taylor's Method Solve $\frac{dy}{dx} = x^2 - y$ with y(0)=1. Also find y at x = 0.1 (6)

c. Find the Volume of the Tetrahedron bounded by the planes x = 0, y = 0, z = 0 & x+y+z = a (8)

Q.6.a. In a single closed circuit, the current i at any time t, is given by $Ri + L\frac{di}{dt} = E$. (6)

Find the current i at a time t if at t = 0, i = 0 and L, R, E are constants.

b. Find the mass of the octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, *the density at any point being kxyz.* (6)

c. Using Runge kutta 's Fourth order method find y at x = 0.2 if $\frac{dy}{dx} = x + y^2$ given that y=1 when x = 0 in steps of h=0.1. (8)

Q.7.a. State and prove Duplication formula for gamma functions. (6)

b. Find the length of the cardiode r = a (1 + cos θ) which lies outside the circle r + a cos = 0 (6)

c. *Solve* : $(1 + 2x)^2 \frac{d^2y}{dx^2} - 6\,(1+ 2x)\frac{dy}{dx} + 16\,y = 8\,(1 + 2x)^2$ (8)

SA 1st half 90

Con. 3432-12. 18/5/2012 FE | Sem-II (REV) Applied maths-II GN-4865

(3 Hours) [Total Marks : 100

N.B. : (1) Question **No. 1** is **compulsory.**
(2) Attempt any **four** questions from the remaining **six** questions.
(3) Figures to the **right** indicate **full** marks.

Q1.a) Evaluate $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta$ (20)

b) Solve $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$

c) Show that $\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)}dx = \frac{\pi}{2}\log(1+a)$

d) Change the order of integration $\int_0^1 \int_{2y}^{2(1+\sqrt{1-y})} f(x,y)\,dx\,dy$

Q2a) Solve $(D-1)^2(D^2+1)y = e^x + \sin^2\left(x/2\right)$ (06)

b) Show that $\int_0^{\infty} xe^{-x^8}dx \int_0^{\infty} x^2 e^{-x^4}dx = \frac{\pi}{16\sqrt{2}}$ (06)

c) Using Runge-Kutta 4th order method find an approximate value of y given that (08)

$\frac{dy}{dx} = x + y^2$ with $x_0 = 0, y_0 = 1$ at $x = 0.1$ *and* $x = 0.2$

Q3 a) In a circuit containing inductance L, resistance R, voltage E, the current I is (06)

given by $L\frac{dI}{dt} + RI = E$ .Find the current I at time t if at t=0, I=0 and L, R, E

are constant.

b) Find the area common of the circles $r = a$ and $r = 2a\cos\theta$ (06)

c) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = \cos\log x + x\sin\log x$ (08)

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Q4 a) Find the volume bounded by $y^2 = x, x^2 = y$ and the plane $z = 0$ and $x + y + z = 2$ (06)

b) Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$ (06)

c) Solve by method of variation of parameters $(D^2 - 3D + 2)y = \frac{e^x}{1+e^x}$ (08)

Q5a) Using Euler's method find the approximate value of y where $\frac{dy}{dx} = x + y, y(0) = 1$ (06)

taking h=0.2 at x=1.

b) A lamina is bounded by $y = x^2 - 3x, y = 2x$ .If the density at any point is given by $\frac{24}{25}xy$. (06)

Find the mass of lamina.

c) Change the order of integration and evaluate $\int_0^{a}\int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$ (08)

Q6 a) Change to polar coordinates and evaluate $\int_0^{\frac{a}{\sqrt{2}}}\int_y^{\sqrt{a^2-y^2}} \log(x^2 + y^2)\,dx\,dy$ (06)

b) Find the length of the cardiode $r = a(1 + \cos\theta)$ which lies out side the circle (06)

$r + a\cos\theta = 0$

c) State Duplication formula of Gamma Function and prove that

$\beta(n,n)\times\beta\left(n+\frac{1}{2}, n+\frac{1}{2}\right) = \frac{\pi}{n}\ 2^{1-4n}$ (08)

Q7. a) Find the volume bounded by cylinder $y^2 + x^2 = 4$ and the plane $z = 0$ (06)

and $y + z = 4$

b) Solve $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$ (06)

c) Solve $(D^4 + 2D^2 + 1)y = x^2\cos x$ (08)