F.E. CBGS NOV. 13 sem II
sub:- maths. 21/11/13

13-11-13-DTP7-RM-1

Con. 8055-13. **GX-10120**

(Revised course)

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Answer any **three** questions from question nos. 2 to 6.
(3) **Figures** to the **right** indicate **full** marks.
(4) Programming Calculators are **not allowed**.

1. (a) Evaluate $\int_0^{\infty} x^2 7^{-4x^2}\, dx$ — 3
   (b) Solve $(D^4+4)y = 0$ — 3
   (c) Prove that $E\nabla = \Delta = \nabla E$ — 3
   (d) Solve $(x + 2y^3)\dfrac{dy}{dx} = y$. — 3
   (e) Evaluate $\iint_R r^3\, dr d\theta$ over the region between the circles $r = 2\sin\theta$, $r = 4\sin\theta$. — 4
   (f) Evaluate $\int_0^1 \int_y^{\sqrt{y}} \dfrac{x}{(1-y)\sqrt{y-x^2}}\, dy dx$ — 4

2. (a) Solve :- $(x^3y^4 + x^2y^3 + xy^2 + y)\, dx + (x^4y^3 - x^3y^2 - x^2y + x)dy = 0$ — 6
   (b) Change the order of integral and hence evaluate $\int_0^5 \int_{2-x}^{2+x} dxdy$ — 6
   (c) Prove that $\int_0^1 \dfrac{x^2}{\sqrt{1-x^4}} dx \int_0^1 \dfrac{dx}{\sqrt{1+x^4}} = \dfrac{\pi}{4\sqrt{2}}$ — 8

3. (a) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dxdydz$. — 6
   (b) Find the area of one loop of the lemniscate $r^2 = a^2 .\cos 2\theta$ — 6
   (c) Solve $(D^3+2D^2+D)y = x^2e^{3x} + \sin^2 x + 2^x$. — 8

4. (a) Show that the length of arc of the parabola $y^2 = 4ax$ cut off by the line $3y = 8x$ is $a\left(\log 2 + \dfrac{15}{16}\right)$ — 6
   (b) Using the method of variation of parameters solve $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$. — 6
   (c) Compute $y(0.2)$ given $\dfrac{dy}{dx} + y + xy^2 = 0$, $y(0) = 1$ by taking $h = 0.1$ using Runge-Kutta method of fourth order correct to 4 decimals. — 8

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5. (a) Solve $\frac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1$. **6**

(b) Solve $\frac{dy}{dx} - 2y = 3e^x$, $y(0) = 0$ using Taylor series method. Find approximate value of y for x = 1 and 1.1. **6**

(c) Evaluate $\int_0^6 \frac{dx}{1+x}$ using **8**

(i) Trapezoidal rule

(ii) Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule and

(iii) Simpson's $\left(\frac{3}{8}\right)^{th}$ rule.

Compare result with exact values.

6. (a) The current in a circuit containing an inductance L, registance R and voltage E sin wt is given by **6**

$$L\frac{di}{dt} + Ri = E \sin wt$$

If i = 0 at t = 0, find i.

(b) Evaluate $\iint_R e^{2x-3y}\, dxdy$ over the triangle bounded by x + y = 1, x = 1, y = 1. **6**

(c) (i) Find the volume of solid bounded by the surfaces $y^2 = 4ax$, $x^2 = 4ay$ and the planes Z = 0, Z = 3. **4**

(ii) Change to polar co-ordinates and evaluate **4**

$$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dxdy}{\sqrt{a^2-x^2-y^2}}$$