(REVISED COURSE)

GS-5427

(3 Hours)

Total Marks: 80

N.B.: (1) Question No. 1 is compulsory.

- Answer any three questions from Question Nos. 2 to 6.
- Figures to the right indicate full marks.
- Programmable calculators are not allowed.

1. (a) Evaluate
$$\int_{0}^{1} (x \log x)^{4} dx$$

- (b) Solve $(D^2 1) (D 1)^2 y = 0$.
- (c) prove that $E = 1 + \Delta = e^{hD}$.
- (d) Solve $\frac{dy}{dx} = \frac{y+1}{(y+2)e^y x}$
- (e) Change into Polar co-ordinates and Evaluate $\int_{a}^{a} \int_{a}^{\sqrt{a^2-x^2}} (x^2+y^2) dy dx.$

(f) Evaluate
$$\int_{0}^{1} \int_{0}^{\sqrt{1-x^2}} \frac{dx \, dy}{1+x^2+y^2}$$

2. (a) Solve $(x^3 y^3 - xy) dy = dx$.

- (b) Change the order of Integration and Evaluate $\int_{a}^{1} \int_{a}^{2-x} \frac{x}{y} \, dy \, dx$.
- (c) (i) P.T. $\int_{0}^{\pi/2} \tan^{n} x \, dx = \frac{\pi}{2} \sec \left[\frac{n\pi}{2} \right].$

(ii) Evaluate
$$\int_{0}^{\infty} \frac{\log(1+ax^2)}{x^2} dx, a > 0$$

- 3. (a) Evaluate $\int_{0}^{1} \int_{0}^{1-x} \int_{0}^{1-x-y} \frac{dz \, dy \, dx}{(1+x+y+z)^{3}}.$
 - (b) Find the area using Double integration where the region of integration is bounded by the curves 9xy = 4 and 2x + y = 2.

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(c) (i) Solve
$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 4y = \cos(\log y)$$
.

(ii) Solve the equation by method of variation of parameters

 $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^X}.$

- 4. (a) Show that for the parabola $r = \frac{2a}{1 + \cos\theta}$ for $\theta = 0$ to $\frac{\pi}{2}$ is $a\left[\sqrt{2} + \log\left(1 + \sqrt{2}\right)\right]$.
 - (b) Solve $\frac{d^2y}{dx^2} + 2y = x^2 e^{3x} + e^x \cos 2x$.
 - (c) Apply Runge-Kutta method of fourth order to find an approximation value of y at x = 0.2 if $\frac{dy}{dx} = x + y^2$ given y = 1 when x = 0 in steps of h = 0.1.
- 5. (a) Solve $(2xy^4e^y + 2xy^3 + y) dx + [x^2y^4e^y x^2y^2 3x] dy = 0$.
 - (b) Solve $\frac{dy}{dx} = 2x + y$ with initial conditions $x_0 = 0$, $y_0 = 0$ by Taylor's method obtain y as series in powers of x.

Find approximation value of y for x = 0.2, 0.4. Compare your result with exact values.

- (c) Evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ by:
 - (i) Trapizoidal method (ii) Simpson's $\frac{1}{3}^{rd}$ method and (iii) Simpsons $\frac{3}{8}^{th}$ method. Compare result with exact values.
- 6. (a) In a circuit containing inductance L, resistance R and voltage E. The current i is given by
 L di/dt + Ri = E. Find current i at time t if t = 0, i = 0 and L, R, E are constants.
 - (b) Evaluate $\iint_{R} xy \, dx \, dy$ where R is the region bounded by $x^2 + y^2 2x = 0$, y = x and $y^2 = 2x$.
 - (c) (i) Find volume of tetrahedron bounded by plane x = 0, y = 0, z = 0 and x + y + z = a.
 - (ii) Find volume bounded by cone $z^2 = x^2 + y^2$ and Paraboloid $z = x^2 + y^2$.