

1. (a) If  $y = \frac{8x}{x^3 - 2x^2 - 4x + 8}$  find  $y_n$  5
- (b) If  $|z^2 - 1| = |z|^2 + 1$  prove that  $z$  lies on imaginary axis where  $z$  is a complex number. 5
- (c) If  $u = \log(x^3 + y^3 - x^2y - xy^2)$  prove that— 5

$$\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = \frac{-4}{(x+y)^2}$$

- (d) Evaluate  $\lim_{x \rightarrow a} \left[ \frac{1}{2} \left( \sqrt{\frac{a}{x}} + \sqrt{\frac{x}{a}} \right) \right]^{x-a}$  5

2. (a) Prove that  $\cos^6 \theta - \sin^6 \theta = \frac{1}{16} [\cos 6\theta + 15 \cos 2\theta]$ . 6
- (b) Test the convergence of— 6

$$\left( \frac{2^2}{1^2} - \frac{2}{1} \right)^{-1} + \left( \frac{3^3}{2^3} - \frac{3}{2} \right)^{-2} + \left( \frac{4^4}{3^4} - \frac{4}{3} \right)^{-3} + \dots$$

- (c) If  $f, g$  and  $h$  are continuous on  $[a, b]$  and differentiable on  $(a, b)$  prove that there exist  $c \in (a, b)$  such that— 8

$$\begin{vmatrix} f'(c) & g'(c) & h'(c) \\ f(a) & g(a) & h(a) \\ f(b) & g(b) & h(b) \end{vmatrix} = 0$$

Deduce Cauchy's and Lagrange's mean value Theorem from this result.

3. (a) If  $x + iy = c \cot(u + iv)$  show that— 6

$$\frac{x}{\sin(2u)} = \frac{-y}{\sinh(2v)} = \frac{c}{\cosh(2v) - \cos(2u)}$$

- (b) If  $v = \log \sin \left[ \frac{\pi(2x^2 + y^2 + xz)^{\frac{1}{2}}}{2(x^2 + xy + 2yz + z^2)^{\frac{1}{3}}} \right]$  show that— 6

$$x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} + z \frac{\partial v}{\partial z} = \frac{1}{3} e^{-v} \sqrt{1 - e^{2v}} \sin^{-1}(e^v)$$

- (c) If  $y = \frac{x}{x^2 + a^2}$  prove that  $y_n = \frac{(-1)^n n!}{a^{n+1}} \sin^{n+1} \theta \cos(n+1)\theta$ . 8

4. (a) Considering only principle value, if  $(1 + i \tan \alpha)^{1 + i \tan \beta}$  is real prove that its value is  $(\sec \alpha)^{\sec^2 \beta}$ . 6

- (b) Show that  $x \operatorname{cosec} x = 1 + \frac{x^2}{6} + \frac{7}{360} x^4 + \dots$  6

- (c) If  $\vec{r} = xi + yj + zk$  and  $\vec{a}, \vec{b}$  are constant vectors, prove that 8

$$\vec{a} \cdot \nabla \left( \vec{b} \cdot \frac{1}{r} \right) = 3 \frac{(\vec{a} \cdot \vec{r})(\vec{b} \cdot \vec{r})}{r^5} - \frac{\vec{a} \cdot \vec{b}}{r^3}$$

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5. (a) Find the directional derivative of  $\phi = x^2 + y^2 + z^2$  in the direction of the line 6

$$\frac{x}{3} = \frac{y}{4} = \frac{z}{5} \text{ at } (1, 2, 3).$$

- (b) If  $u = f(2x - 3y, 3y - 4z, 4z - 2x)$  6

Show that—  $6 \frac{\partial u}{\partial x} + 4 \frac{\partial u}{\partial y} + 3 \frac{\partial u}{\partial z} = 0$

- (c) Find a point in the plane  $x + 2y + 3z = 13$  nearest to the point  $(1, 1, 1)$  using the method of Lagrange's multipliers. 8

6. (a) Show that  $\sin^{-1}(x) = x + \frac{x^3}{6} + \frac{3}{40}x^5 + \dots$  6

- (b) State and prove Euler's theorem for two variables. 6

- (c) Prove that nth root of unity are in geometric progression. Also find sum of nth root of unity. 8

7. (a) Find  $(1.04)^{3.01}$  by using theory of approximation. 6

- (b) If  $y^{1/m} + y^{-1/m} = 2x$  prove that — 6

$$(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0.$$

- (c) If  $u = \log \tan \left( \frac{\pi}{4} + \frac{\theta}{2} \right)$  prove that— 8

(i)  $\cos h(u) = \sec(\theta)$

(ii)  $\sin h(u) = \tan \theta$

(iii)  $\tan h(u) = \sin \theta$

(iv)  $\tan h(u/2) = \tan(\theta/2)$ .