

Con. 5543-10.

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(3 Hours)

[Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.

(2) Attempt any **four** from remaining **six** questions.

(3) Figures to the **right** indicate **full** marks.

(4) Assume the **suitable** data if needed with **justification**.

1. (a) Express $(1 + 7i)(2 - i)^{-2}$ in the form of $r(\cos \theta + i \sin \theta)$ and prove that its fourth power is a real negative number. 5

(b) If $u = z \tan^{-1} \frac{y}{x}$ find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ 5

(c) Find $\nabla \cdot \vec{F}$ & $\nabla \times \vec{F}$ where $\vec{F} = \frac{1}{(x^2 + y^2)}(x\hat{i} - y\hat{j})$ 5

(d) Test the convergence of the series $\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$ 5

2. (a) Prove that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$
Hence expand $\log(1 + x^2)$ in powers of x 8

(b) If $z = f(x, y), x = u \cosh v, y = u \sinh v$, prove that $\left(\frac{\partial z}{\partial u}\right)^2 - \frac{1}{u^2} \left(\frac{\partial z}{\partial v}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2$ 6

(c) Show that for real values of a and b , $e^{2a \cot^{-1} b} \left[\frac{b-1}{b+1}\right]^{-a} = 1$ 6

3. (a) State Euler's theorem for a function of 3 variables and hence verify the same for $u = \log \left(\frac{xy+yz+zx}{x^2+y^2+z^2}\right)$ 8

(b) Determine $y_n(0)$ if $y = \frac{x^3}{x^2-1}$ 6

(c) Find the constants a, b, c if the normal to the surface $ax^2 + yz + bxz^3 = c$ at $P(1,2,1)$ is parallel to the normal to the surface $y^2 + xz = 61$ at $Q(10,1,6)$. 6

4.(a) Find the cube roots of unity. If w is a complex cube root of unity, prove that

(i) $1+w+w^2=0$

(ii) $\frac{1}{1+2w} + \frac{1}{2+w} - \frac{1}{1+w} = 0$

8

(b) Use Rolle's Theorem to prove that the equation $ax^2 + bx = \frac{a}{3} + \frac{b}{3}$ has a root between 0 & 1.

6

(c) Find $xu_x + yu_y + zu_z$ for $u = \cos \frac{xy + yz}{x^2 + y^2 + z^2} + \sin(\sqrt{x} + \sqrt{y} + \sqrt{z})$

6

5.(a) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$,

prove that $(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2 + (\frac{\partial u}{\partial z})^2 = 2(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z})$

8

(b) Find a, b, c given that $\lim_{x \rightarrow 0} \frac{ae^x - be^{-x} - cx}{x - \sin x} = 4$

6

(c) Prove that $\sinh^{-1}(\tan \theta) = \log \left(\tan \left(\frac{\theta}{2} + \frac{\pi}{4} \right) \right)$.

6

6.(a) if $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$,

Prove that $(x^2 - 1)y_{n+2} + x(2n + 1)y_{n+1} + (n^2 - m^2)y_n = 0$

8

(b) The diameter and altitude of a can in shape of right circular cylinder are measured as 4cm & 6cm respectively. The possible error in each measurement is 0.1cm. Find approximately the maximum possible error in the value computed for the volume & lateral surface.

6

(c) If $x + iy = c \cot(u + iv)$, show that $\frac{x}{\sin 2u} = \frac{-y}{\sinh 2v} = \frac{c}{\cosh 2v - \cos 2u}$

6

7.(a) Find the stationary values of $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

8

(b) If $a = \cos 3\alpha + i \sin 3\alpha, b = \cos 3\beta + i \sin 3\beta, c = \cos 3\gamma + i \sin 3\gamma$ then prove that

$$\sqrt[3]{\frac{ab}{c}} + \sqrt[3]{\frac{c}{ab}} = 2 \cos(\alpha + \beta - \gamma)$$

6

(c) if $\vec{r} = a \cos t \hat{i} + a \sin t \hat{j} + at \tan \alpha \hat{k}$, then prove that

(i) $[\dot{\vec{r}} \ \ddot{\vec{r}} \ \dddot{\vec{r}}] = a^3 \tan \alpha$

(ii) $|\dot{\vec{r}} \times \ddot{\vec{r}}| = a^2 \sec \alpha$

6