

Con. 2648-10.

(REVISED COURSE)

AN-9752

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(3 Hours)

[Total Marks : 100

N.B. (1) Question No. 1 is compulsory.

(2) Attempt any four questions from remaining six questions.

(3) Figures to the right indicate the full marks.

(4) Assume the suitable data if needed with justification.

1. (a) If $|z - 1| < |z + 1|$ prove that $\text{Re } z > 0$ 5
 (b) If $u = \log (\tan x + \tan y + \tan z)$. Prove that— 5

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2.$$

- (c) Find $\phi(r)$ s.t. $\nabla \phi = \frac{\vec{r}}{r^5}$ and $\phi(1) = 0$. 5

- (d) Prove that : $e^{x \cos x} = 1 + x + \frac{x^2}{2} + \dots$ 5

2. (a) If $x + \frac{1}{x} = 2 \cos \theta$, $y + \frac{1}{y} = 2 \cos \phi$ then show that— 6

$$x^2 y^2 + \frac{1}{x^2 y^2} = 2 \cos (2\theta + 2\phi)$$

- (b) Prove : $\tan^{-1} \left(\frac{\sqrt{1+x^2} - 1}{x} \right) = \frac{1}{2} \left(x - \frac{x^3}{3} + \frac{x^5}{5} - \dots \right)$ 6

- (c) Find all the stationary points of $f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ 8
 Determine which are maximum and minimum.

3. (a) If a, a^2, a^3, \dots, a^6 are the roots of $x^7 - 1 = 0$ prove that $(1 - a)(1 - a^2) \dots (1 - a^6) = 7$ 6

- (b) Test the convergence of — 6

$$\frac{3+4}{4+5}, \frac{3^2+4^2}{4^2+5^2}, \frac{3^3+4^3}{4^3+5^3}, \dots$$

- (c) Verify : $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{b} \cdot \vec{c}) \vec{a}$ and 8

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$$

For $\vec{a} = 3i - 2j + 2k$; $\vec{b} = 6i + 4j + 2k$; $\vec{c} = 3i + 2j + 4k$

4. (a) If $x = e^u \tan v$, $y = e^u \sec v$ find, $\left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \right) \cdot \left(x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} \right)$ 6

(b) Prove that the equation $2x^3 - 3x^2 - x + 1 = 0$ has one root between 1 and 2. 6

(c) Show that $\log(e^{i\alpha} + e^{i\beta}) = \log 2 \cos\left(\frac{\alpha - \beta}{2}\right) + i\left(\frac{\alpha + \beta}{2}\right)$. 8

5. (a) State and prove Euler's Thm. For function of two variables. 6

(b) If $\sin(\theta + i\phi) = \tan \alpha + i \sec \alpha$ then show that — 6
 $\cos 2\theta \cosh 2\phi = 3$.

(c) If $y = \cos^{-1} x$ prove that, 8
 $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0$

6. (a) If $y = \frac{\log x}{x}$ prove that 6

$$y_5 = \frac{5!}{x^6} \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \log x \right]$$

(b) Find the product of all values of $\left(\frac{1}{2} - \frac{i\sqrt{3}}{2} \right)^{\frac{3}{4}}$ 6

(c) If $u = x^3 \sin^{-1} \frac{y}{x} + x^4 \tan^{-1} \frac{y}{x}$ find the value of, 8

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \text{ at } x=1 \text{ and } y=1$$

7. (a) Find 'a', 'b', 'c' if $\lim_{x \rightarrow 0} \frac{x(a + b \cos x) - c \sin x}{x^5} = 1$ 6

(b) State the Lagrange's Thm and give its geometrical interpretation. 6

(c) If $u = x^y$ show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$. 8