

- N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions from remaining **six**.

1. (a) Prove that $\int_{\frac{3}{2}-x}^{\frac{3}{2}+x} \left(\frac{1}{4} - x^2\right) \pi \sec \pi x$ provided $-1 < 2x < 1$. 5
- (b) Solve $(x^2 + y^2 + 1) dx - 2xydy = 0$. 5
- (c) Show that $\int_0^{\infty} \frac{\log(1+ax^2)}{x} dx = \pi\sqrt{a}$ ($a > 0$). 5
- (d) Find the total length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$. 5
2. (a) Prove that $\int_0^{\pi} \frac{\sin^{n-1} x}{(a+b\cos x)^n} dx = \frac{2^{n-1}}{(a^2-b^2)^{n/2}} \beta\left(\frac{n}{2}, \frac{n}{2}\right)$ 6
- (b) Change the order of integration and evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{\cos^{-1} x}{\sqrt{1-x^2} \sqrt{1-x^2-y^2}} dx dy$. 6
- (c) Solve $(D^3 + 1)y = e^{\frac{x}{2}} \sin\left[\frac{\sqrt{3}}{2} x\right]$. 8
3. (a) Find by double integration the area enclosed by the curve $9xy = 4$ and the line $2x + y = 2$. 6
- (b) Evaluate $\iint_R \frac{dx dy}{x^4 + y^2}$ where R is the region $x \geq 1$ and $y \geq x^2$. 6
- (c) Solve $(D^2 - D)y = e^x \sin x$ by method of undetermined coefficient. 8
4. (a) Find the volume bounded by $y^2 = x$, $x^2 = y$ and the planes $z = 0$ and $x + y + z = 1$. 6
- (b) Evaluate $\int_0^{\infty} x^{-x^8} e^{-x} dx$ $\int_0^{\infty} x^2^{-x^4} e^{-x} dx$. 6
- (c) Solve $(D^2 - 1)y = 2(1 - e^{-2x})^{\frac{-1}{2}}$ by variation of parameter. 8

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5. (a) Find the mass of the lamina bounded by the curve $ay^2 = x^3$ and the line $y = x$. If the density at a point varies as the distance of the point from the x axis. 6

(b) Verify the rule of D.U.I.S. for $\int_0^{\infty} e^{-at} \sin bt dt$. 6

(c) Solve $\left(\frac{d}{dx} + \frac{1}{x}\right)^2 y = \frac{1}{x^4}$. 8

6. (a) Evaluate $\iiint x^2 y z dx dy dz$ throughout the volume bounded by $x = 0$ $y = 0$ $z = 0$ and 6

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

(b) Find the length of the upper arc of one loop of lemniscate $r^2 = a^2 \cos 2\theta$. 6

(c) Solve $(1+x)^2 \frac{d^2 y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x)$. 8

7. (a) Change to polar co-ordinates and evaluate $\iint_R \frac{1}{\sqrt{xy}} dx dy$ where R is the region of 6

Integration bounded by $x^2 + y^2 - x = 0$.

(b) Solve $\frac{dy}{dx} = x^3 y^3 - xy$. 6

(c) Prove that $\beta\left(n + \frac{1}{2}, n + \frac{1}{2}\right) = \frac{1}{2^{2n}} \frac{\sqrt{\pi}}{\Gamma(n+1)}$ 8

deduce that $2^n \sqrt{n + \frac{1}{2}} = 1.3.5 \dots (2n-1) \sqrt{\pi}$.