

- N.B. : (1) Question No. 1 is **compulsory**.  
(2) Attempt any **four** questions out of remaining **six** questions.  
(3) **Figures** to the **right** indicates **full** marks.

1. (a) Show that  $\int_0^1 \sqrt{1-x^4} dx = \frac{\sqrt{\pi}}{6} \left[ \frac{1}{4} \right]$  5

(b) Find the total length of the curve  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$  5

(c) Change the order of integration and evaluate  $\int_0^5 \int_{2-x}^{2+x} dx dy$  5

(d) Solve  $6 \frac{d^2y}{dx^2} + 17 \frac{dy}{dx} + 12y = e^{-\frac{3x}{2}} + 2^x$  5

2. (a) Evaluate  $\int_0^{\pi} \frac{dx}{a+b\cos x}$ ,  $a > 0$ ,  $b > 0$  and deduce 8

that  $\int_0^{\pi} \frac{dx}{(a+b\cos x)^2} = \frac{\pi a}{(a^2 - b^2)^{3/2}}$

(b) Show that  $\int_0^a \frac{dx}{(a^n - x^n)^{1/n}} = \frac{\pi}{n} \operatorname{cosec}\left(\frac{\pi}{n}\right)$  6

(c) Evaluate  $\iiint x^2 y z dx dy dz$  throughout the volume bounded by the planes 6  
 $x = 0, y = 0, z = 0, \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$

3. (a) Solve  $\left(\frac{y}{x} \sec y - \tan y\right) dx - (x - \sec y \log x) dy = 0$  6

(b) Using Eulers Method find approximate value of  $y$  at  $x = 1$  in five steps taking 6  
 $h = 0.2$ , Given  $\frac{dy}{dx} = x + y$  and  $y(0) = 1$

(c) Solve  $(D^3 + D) y = \operatorname{cosec} x$ , by method of variation of parameters. 8

4. (a) Solve by Runge-Kutta Method of fourth order  $\frac{dy}{dx} = 3x + y^2$ ,  $x_0 = 1$ ,  $y_0 = 1.2$  at  $x = 1.1$ . 8
- (b) Solve by Taylors Series Method  $\frac{dy}{dx} = -xy$  with  $x_0 = 0$ ,  $y_0 = 1$ . 6
- (c) Solve  $(x+2)^2 \frac{d^2v}{dx^2} - (x+2) \frac{dv}{dx} + v = 3x+4$ . 6

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5. (a) Find the area of the Cardioide,  $r = a(1 + \cos \theta)$ . 6
- (b) Solve  $\frac{dy}{dx} \cosh x = 2 \cosh^2 x \sinh x - y \sinh x$ . 6
- (c) (i) Solve  $\frac{dy}{dx} = 1 - x(y - x) - x^3(y - x)^2$  4
- (ii) Solve  $\frac{d^4 y}{dx^4} - a^4 y = \sin ax$ . 4
6. (a) Evaluate  $\iint_R xy(x+y) dx dy$  where R is the region bounded by  
 $xy = 4, y = 0, x = 1, x = 4$ . 6
- (b) Evaluate  $\int_0^{\pi/4} \int_0^{\sqrt{\cos 2\theta}} \frac{r}{(1+r^2)^2} d\theta dr$  6
- (c) Solve  $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = xe^x \sin x$ . 8
7. (a) Evaluate  $\int_{-\pi}^{\pi} \sin^2 x \cos^4 x dx$  6
- (b) (i) The differential equation of motion of a body is  $\frac{d^2 x}{dt^2} + n^2 x = t \cos t$  4  
 Solve this equation, what is the solution if  $i = n^2$ .
- (ii) The density of a uniform circular lamina of radius 'r' varies as the square of its distance from a fixed point on the circumference of the circle. 4  
 Find the mass of the lamina.
- (c) Sketch the area of integration and evaluate 6

$$\int_1^2 \int_{-\sqrt{2-y}}^{\sqrt{2-y}} 2x^2 y^2 dx dy$$