

Library)
Con. 3706-09.

(REVISED COURSE)

10.30 to 1.30

SP-8445

(3 Hours)

[Total Marks : 100

N.B.: (1) Question No. 1 is **compulsory**.(2) Attempt any **four** questions from the remaining **six** questions.(3) **Figures to the right indicate full marks.**

1. (a) Solve $y(x^2y^2 + 2) dx + x(2 - 2x^2y^2) dy = 0$. 5

(b) Find area of the cardioid $r = a(1 + \cos\theta)$ lying outside the circle $r = \frac{3}{2}a$. 5

(c) Show that $\int_0^2 (8 - x^3)^{-1/3} dx = \frac{2\pi}{3\sqrt{3}}$. 5

(d) Prove that $\int_0^1 \frac{x^a - x^b}{\log x} dx = \log\left(\frac{a+1}{b+1}\right)$, using D.U.I.S. rule. 5

2. (a) Solve $\frac{dy}{dx} + \frac{y \log y}{x - \log y} = 0$ 6

(b) Show that $\int_0^\infty x^{m-1} \cos(ax) dx = \frac{\Gamma(m)}{a^m} \cos\left(\frac{m\pi}{2}\right)$ 7

(c) Evaluate $\iiint \frac{dx dy dz}{(x^2 + y^2 + z^2)^{1/2}}$ over the volume bounded by spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2$, $b > a > 0$. 7

3. (a) Evaluate $\iint_R y dx dy$ where R is the region bounded by $y^2 = 4x$ and $x^2 = 4y$. 6

(b) Find the length of one arch of the cycloid $x = a(t - \sin t)$, $y = a(1 - \cos t)$ 7

(c) Use Taylor's series method to find $y(1.1)$, given $\frac{dy}{dx} = xy^{1/3}$; $y(1) = 1$. 7

Obtain solution of the differential equation directly and compare the answer.

4. (a) Solve $(D^2 + 3D + 2)y = e^{-2x} + e^x \cos 2x$.

6

(b) Change the order of integration of $\int_0^1 \int_{-\sqrt{2y-y^2}}^{1+\sqrt{1-y^2}} f(x,y) dx dy$.

7

(c) Using Runge-Kutta method of order four find $y(0.2)$ with $h = 0.1$.

7

Given $\frac{dy}{dx} = \frac{1}{x+y}$; $y(0)=1$.

5. (a) Solve $x^3 \frac{d^3 y}{dx^3} + 3x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = x + \log x$

6

(b) Change to polar co-ordinates and evaluate

7

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{xy dx dy}{\sqrt{x^2+y^2}}$$

(c) Find y when $x = 0.05$ by Euler's modified method, taking $h = 0.05$, given that

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$\frac{dy}{dx} = x^2 + y$; $y(0) = 1$.

6. (a) Solve $(xy^3 + y) dx + 2(x^2y^2 + x + y^4) dy = 0$

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(b) Solve $(D^2 + 1)y = \cos ecx \cot x$ using Variation of Parameter method.

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(c) Change the order of integration and evaluate $\int_0^{\infty} \int_0^x x e^{\frac{-x^2}{y}} dy dx$.

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7. (a) The differential equation for electric charge Q of an electric circuit, consisting of an inductance L , capacitance C and an alternating e.m.f. $E \sin(nt)$, applied

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in series is $L \frac{d^2 Q}{dt^2} + \frac{1}{C} Q = E \sin(nt)$. Solve the differential equation to find the charge Q .

(b) Find mass of the lamina bounded by $x^2 + 2y - 4 = 0$ and X axis if the density at any point varies as its distance from X axis.

7

(c) Find the volume of the paraboloid $x^2 + y^2 = 4z$ cut off by the plane $z = 4$.

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