3-p3-upq-Con No. File

29/11/2010
(Exam. Nov. 2010)

FE Sem II
Applied Maths – II

Con. 5544-10.

GT–7845

(3 Hours)

[Total Marks : 100

N.B. i) Question No. 1 is compulsory.

ii) Attempt any four out of remaining six questions.

iii) Figures to the right indicate full marks.

iv) Answers to the individual questions must be grouped and written together.

1. (a) Prove that $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}\,dxdy = \frac{B(m,n)}{a^n b^m}$ (5)

(b) Evaluate by changing to polar co-ordinates $\int_0^1 \int_0^x (x+y)\,dxdy$ (5)

(c) Use differentiation under integral sign to prove that

$\int_0^\infty \frac{\log(1+ax^2)}{x^2}\,dx = \pi\sqrt{a}$, (a>0) (5)

(d) Solve $\frac{dy}{dx} = \frac{y+1}{(y+2)e^y - x}$ (5)

2. (a) Evaluate $\int_0^a \int_0^x \frac{e^y}{\sqrt{(a-x)(x-y)}}\,dxdy$ (6)

(b) Change the order of integration $\int_0^2 \int_{\sqrt{4-x^2}}^{4-x} f(x,y)\,dxdy$ (7)

(c) Show that the length of an arc of that part of cardioids r=a(1+cosθ) which lies on the side of the line 4r=3asecθ remote from the pole is equal to 4a. (7)

3. (a) Solve (1+siny)$\frac{dx}{dy} = [2y\cos y - x(\sec y + \tan y)]$ (6)

(b) Solve $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$. (7)

(c) Find the area common to the circles r=a and r=2acosθ . (7)

4. (a) Use method of variation of parameters to solve the equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}sec^2x(1 + 2tanx) \quad (6)$$

(b) Solve $(3x+2)^2\frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$ (7)

(c) A triangular prism is formed by the planes whose equations are ay= bx, y=0, x=a ,obtain the volume of this prism between the planes z=0 and the surface z=c + xy . (7)

5. (a) Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x + y + z)dxdydz$ (6)

(b) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + e^x + cos2x$ (7)

(c) Solve $\frac{d^3y}{dx^3} - 3\frac{dy}{dx} + 2y = 2e^x\cos(x/2)$ (7)

6. (a) Using Euler's method, find the approximate value of y, when x=1.5 in five steps, taking h=0.1 Given $\frac{dy}{dx} = \frac{y-x}{\sqrt{xy}}$ and y(1)=2 . (6)

(b) Find the mass of the lamina bounded by the curve $y^2 = ax, x^2 = ay$ where the mass per unit area varies as the square of the distance from the origin. (7)

(c) Evaluate $\iint\sqrt{xy(1-x-y)}dxdy$ over the region $x \geq 0, y \geq 0, x + y \leq 1$ (7)

7. (a) Using Taylor's series method solve the equation $\frac{dy}{dx} = 2y + 3e^x$, given $x_0 = 0, y_0 = 1$ at x=0.1 and x=0.2 . (6)

(b) In case of an elastic string which has one end fixed and a particle of mass, m attached to other end, the equation of motion is,

$$m\frac{d^2s}{dt^2} = -\frac{mg}{e}(s - l)$$

, where l is the natural length of the string and e, elongation due to weight mg. Find s such that $s=s_0$, v=0 at t=0. (7)

(c) Show that $\int_0^1 [illegible]\,dx \times \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$ (7)